# Viscosity solutions for realistic Shape-From-Shading

Emmanuel Prados, Olivier Faugeras

# Viscosity solutions
# For
# Realistic Shape-From-Shading

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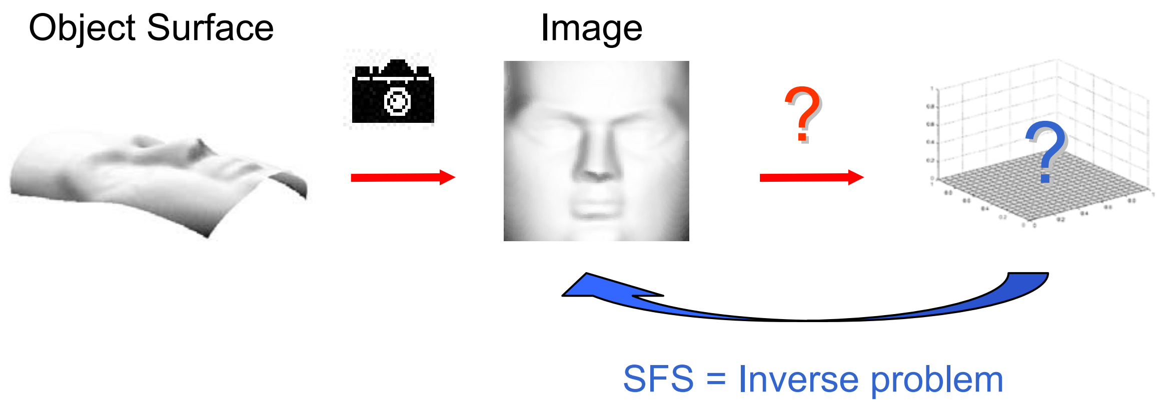
**Workshop “Numerical Methods for Viscosity Solutions and Applications”**
**Rome, September 6-8, 2004**

# The Shape From Shading Problem




**Problem:**
- **Inverse problem to image synthesis,**
- **Recover the surface(s) which yield(s) the same image.**

**Used Information:**
- **Shading.**

# Shape From Shading Data

- One image;
- Intrinsic parameters of the camera:
  - Focal length,
  - Size of the pixels
- Surface parameters:
  - Reflectance of the surface (albedo)
- Lighting parameters:
  - Flash…
- *Database* of *real* images of faces available online:
  - Experimental setup: scene illuminated by the flash of the camera.
  - Parameters are detailed.
  - WWW link:

http://www-sop.inria.fr/odyssee/team/Emmanuel.Prados/

# Report:

1) Most of the SFS methods model the problem very basically:

   Ex.: Camera=Orthographic Projection…

   $\Rightarrow$ Not Realistic!

2) Lots of SFS algorithms require boundary data (generally Dirichlet BC)

   $\Rightarrow$ Not Available with **Real** Images!

$\Rightarrow$ The results on real images are generally very disappointing.

# Shape From Shading Contributions:

- We propose more realistic modeling:

  Example: camera = pinhole

  (perspective projection)

- We reduce as much as possible the requirement of the boundary data

  (in particular on the boundary of the images).

# Modeling / Mathematical formulations

- The Basic Assumption: Lambertian Surface

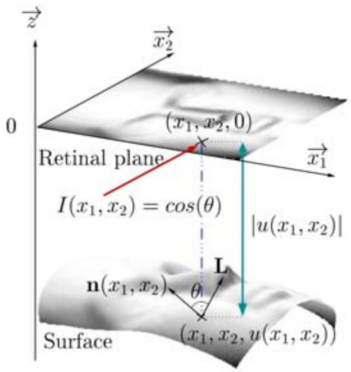
(without specularities or highlights)

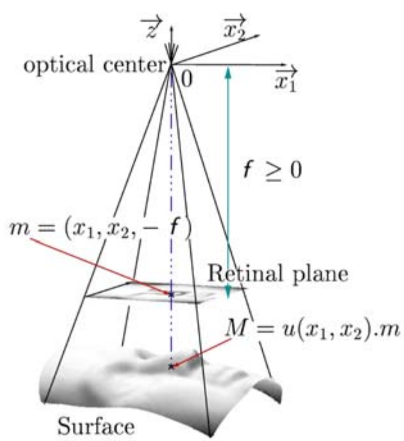
$\Rightarrow$ Involves a connection between:

- the brightness of the image (intensity) : variable $I$,
- the normal surface: $\mathbf{n}$, <<Depends on the parameterization>>.
- the lighting direction: $\mathbf{L}$ <<Depends on the lighting>>.

$$\boxed{I(X) = cos(\mathbf{n}(X), \mathbf{L}(\mathbf{X})) = \frac{\mathbf{n}(X)}{|\mathbf{n}(X)|} \cdot \mathbf{L}(\mathbf{X})}$$

# Modeling / Mathematical formulations




- Orthographic projection (classical modeling)
  - Far light source:

$$I(x)\sqrt{1 + |\nabla u|^2} + \nabla u \cdot \mathbf{l} - \gamma = 0,$$

  - Frontal far light source: (Eikonal equation)

$$|\nabla u| = f(x),$$

$$f(x) = \sqrt{\frac{1}{I(x)^2} - 1},$$

- **More realistic** Modeling: Perspective Projection:
  - Far light source:

$$I(x)\sqrt{f^2|\nabla u|^2 + (x \cdot \nabla u + 1)^2} - (f\mathbf{l} + \gamma x) \cdot \nabla u - \gamma = 0,$$

  - Light source located at the Optical center:

$$I(x)\sqrt{f^2|\nabla u|^2 + (x \cdot \nabla u)^2 + Q(x)^2} - Q(x) = 0.$$

$$Q(x) = \frac{f}{\sqrt{|x|^2 + f^2}}.$$

# A Generic Hamiltonian

- All these SFS equations are special cases of the "generic" equation:

$$\kappa_x \sqrt{|A_x \nabla \mathbf{u} + \mathbf{v_x}|^2 + K_x^2} + \mathbf{w_x} \cdot \nabla \mathbf{u} + c_x = 0 \quad (1)$$

Where $\kappa_x, K_x \geq 0,\ A_x \in \mathcal{M}_{2\times 2},$
$\mu_x, \nu_x \in \mathbb{R}^*,\ \mathbf{v_x}, \mathbf{w_x} \in \mathbb{R}^2$ and $c_x \in \mathbb{R}.$

(1) is a HJ PDE of the form $H(x, \nabla u)$ with:

$$H(x,p) = \kappa_x \sqrt{|A_x p + \mathbf{v_x}|^2 + K_x^2} + \mathbf{w_x} \cdot p + c_x.$$

***H(x,p)*** convex in ***p***.

# Ill-posedness of the SFS problem

- SFS equations are of the form:

$$H(x,\nabla u)=0, \quad x \text{ in } \Omega \quad \text{(a bounded set).}$$

$\Rightarrow$ They require Boundary Conditions (BC) on $\partial\Omega$

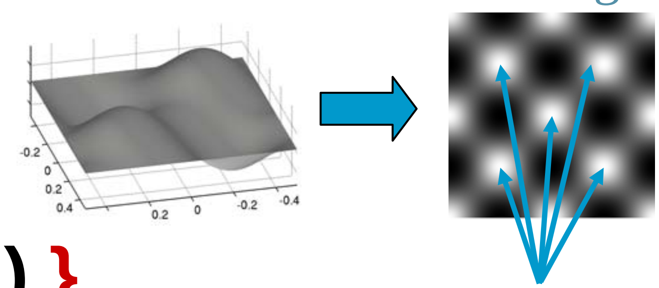
$\Rightarrow$ But the BC on $\partial\Omega$ **are not sufficient!**

**SFS equations can have singularities:**

***For all the previous SFS equations,***

***the set of singular points is***

$$\{\ x \mid I(x) \text{ is maximal (i.e 1)}\ \}$$



# Recall:

In Shape from Shading

**Data = one image.**

$\Rightarrow$ We do not have boundary data;

$\Rightarrow$ i.e. We do not know what are the adequate boundary conditions.

# Solution proposed in the SFS literature

| Notion | SFS Biblio. | Data (DBC) | Limitations |
|---|---|---|---|
| Continuous Viscosity solutions (Crandall-Lions…) | Lions/Rouy/ Tourin (92) | $S_{ing} \cup \partial\Omega$ | ■**Compatibility conditions.** ■**Dirich. data on** $S_{ing} \cup \partial\Omega$ |
| $C^1$ Functions | Dupuis/ Oliensis (94) | Set of local ***Min.*** of $u$ | Value function ? **Few intuitive…** But: Interest : **Minimal Data are required!** |
| Discontinuous Viscosity solutions (Ishii…) | Prados/ Faugeras (03) | $N(S_{ing}) \cup \partial\Omega$ | ■**Dirichlet data on** $N(S)_{ing} \cup \partial\Omega$ |
| Singular Viscosity solutions (Camilli-Siconolfi:99) | Camilli/ Falcone (96) | $\partial\Omega$ | ■ **not always adapted to SFS.** ■ **still requires data on** $\partial\Omega$ |

# Contributions:

- We have slightly extended and modified the notion of singular viscosity solutions (Camilli/Siconolfi:99,02) in order to:
  - to extend the Dupuis/Oliensis' characterization of the $C^1$ solutions to the viscosity solutions $\Rightarrow$ data are minimal.
  - such that compatibility conditions are not required.

# Notion of SDVS

- Definition: We define a notion of solution
  - which extends the classical notions of
    - of the discontinuous viscosity solutions (Ishii…)
    - of the singular viscosity solutions (Camilli-Siconolfi:99,02).
  - Which allows to fix the values of the solutions at the singular points or on the boundary $\partial\Omega$ when we know them, and to send them at the infinity (state constraints), when we do not know them.

# Properties of the SDVS:

- Existence of the SDVS
  - – Dynamical programming principle,
  - – value function;
- Strong uniqueness;
- Stability results;

➡ Application to the Shape From Shading problem…

# Original theorems:

- Theorem: (roughly)

  « A discontinuous viscosity solution without "local minima" is the SDVS».

- Corollary:

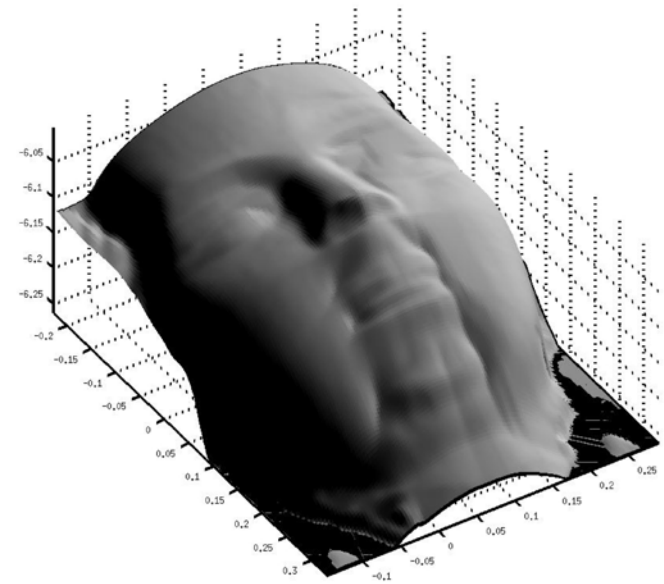
  Characterization of the discontinuous viscosity solutions by their « minima ».

# Numerical approximations:

- 4 stages:
  i) Management of the state constraints;
  ii) Regularization of the equations
  In SFS (we truncate the image, $I_\varepsilon = min(I(x), 1-\varepsilon)$);
  iii) Designing of the approximation scheme;
  iv) Deduction and implementation of the associated algorithm.

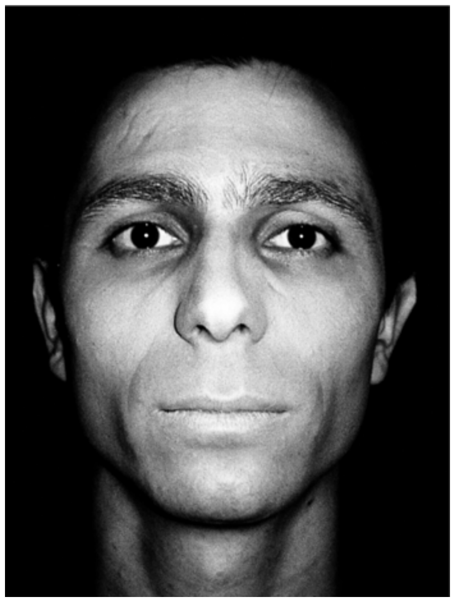
# A Numerical Result on a Synthetic Image (Mozart)

a) Synthetic image of the classical Mozart face

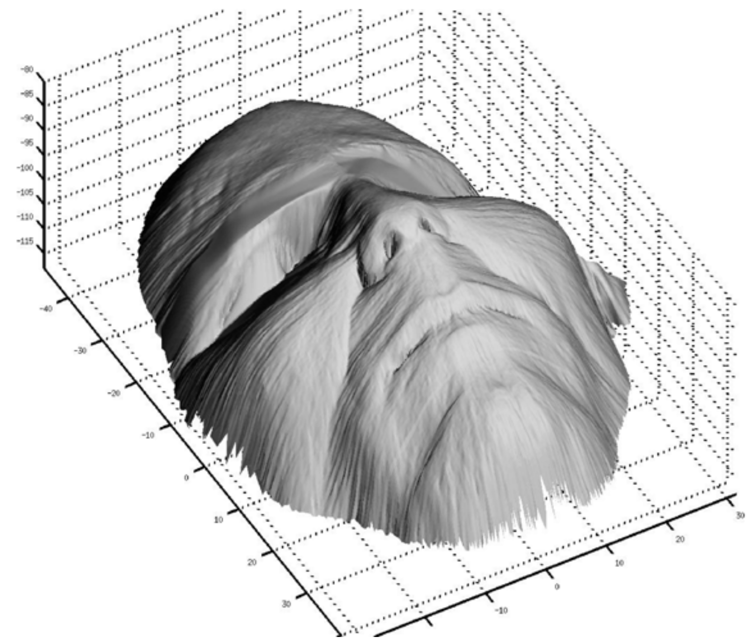


b) Reconstruction

# Numerical results on
# Real images (Faces)…

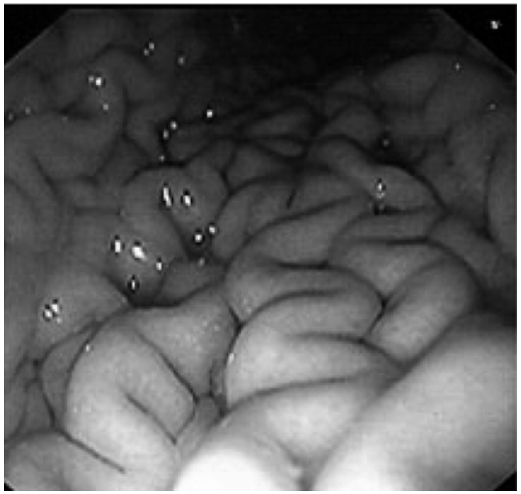


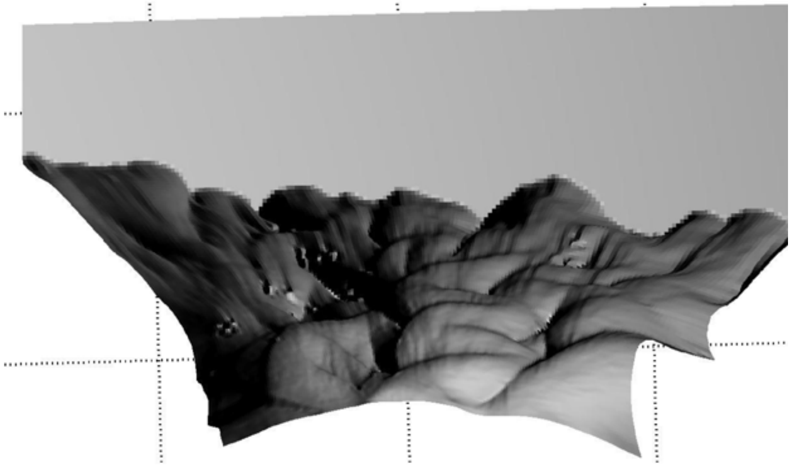
a) Real image of a face



b) Reconstruction

# Numerical results on Real images (Stomach)…

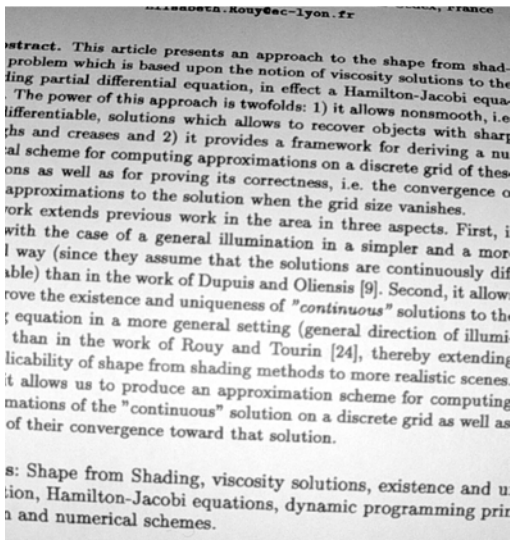




a) Real image of a stomach

b) Reconstruction

# Numerical results on Real images (Pages)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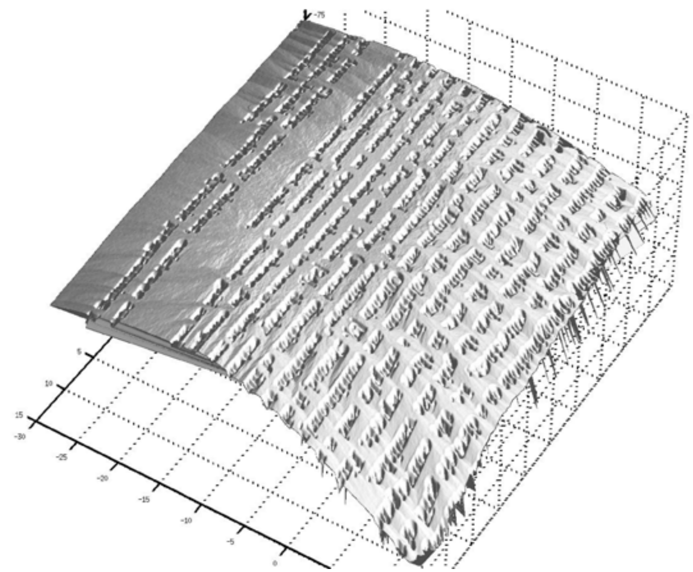
bstract. This article presents an approach to the shape from shad-
problem which is based upon the notion of viscosity solutions to the
ding partial differential equation, in effect a Hamilton-Jacobi equa-
. The power of this approach is twofolds: 1) it allows nonsmooth, i.e
lifferentiable, solutions which allows to recover objects with sharp
ths and creases and 2) it provides a framework for deriving a nu
al scheme for computing approximations on a discrete grid of these
ons as well as for proving its correctness, i.e. the convergence o
approximations to the solution when the grid size vanishes.
ork extends previous work in the area in three aspects. First, i
with the case of a general illumination in a simpler and a mor
l way (since they assume that the solutions are continuously dif
able) than in the work of Dupuis and Oliensis [9]. Second, it allow
rove the existence and uniqueness of "continuous" solutions to th
equation in a more general setting (general direction of illumi
than in the work of Rouy and Tourin [24], thereby extending
licability of shape from shading methods to more realistic scenes
it allows us to produce an approximation scheme for computing
mations of the "continuous" solution on a discrete grid as well as
of their convergence toward that solution.

s: Shape from Shading, viscosity solutions, existence and u
tion, Hamilton-Jacobi equations, dynamic programming prin
n and numerical schemes.



a) Real image of a page

b) Reconstruction

# Conclusion:

- SFS problem ≠ Eikonal equation.
  - Realistic SFS consists in solving more complex equations: for ex. the “generic SFS” equation.
- In SFS, we do not have at our disposal boundary data.
  - The notion of SDVS proposes some more adapted solutions than the other classical solutions.

# For more details:

- **Theoretical part:**

  **See the INRIA research report:**

  **"A viscosity method for Shape From Shading without boundary data"**

  ***(E. Prados, F. Camilli and O. Faugeras), August 2004.***

- **Numerical part: See**
  - **the INRIA research report No RR-5005 :**

    **"A mathematical and algorithmic study of the Lambertian SFS problem for orthographic and pinhole cameras"**

    ***(E. Prados and O. Faugeras)*, September 2003.**
  - **E. Prados' PhD thesis, October 2004.**

- **Web page:**

  http://www-sop.inria.fr/odyssee/team/Emmanuel.Prados/

- **Thanks!**